DOCUMENT RESUME

ED 228 092 SE 041 289

AUTHOR Throm, Elaine G.
TITLE The Pragmatism of Problem Solving in the Mathematical School Curriculum.
PUB DATE Nov 82
NOTE 66p.
PUB TYPE Reference Materials - Bibliographies (131)

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS Annotated Bibliographies; Cognitive Processes; Educational Research; Elementary Secondary Education; Guidelines; *Mathematics Curriculum; *Mathematics Education; *Mathematics Instruction; *Problem Solving; *Reference Materials; Teaching Methods
IDENTIFIERS Mathematics Education Research

ABSTRACT

References on problem solving are categorized and annotated in this document. The philosophy of problem solving, especially its meaning and its implications for the mathematics curriculum, are examined in references included in the first section. In the second section, references on different strategies and techniques of problem solving that teachers can follow are presented. Recommendations are divided by time period (pre-1960, 1960s, 1970s, 1980s). The third section lists references on a variety of applications of problem solving in the typical classroom, with sections for elementary, junior high, and high school teachers. Finally, a summary draws together some of the ideas in the various references, and conclusions and recommendations are briefly stated. A bibliography and additional sources are listed, and appendices contain a flow chart of a problem-solving approach and a conceptual map of problem solving. (MNS)

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THE PRAGMATISM OF PROBLEM SOLVING IN THE MATHEMATICAL SCHOOL CURRICULUM

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November, 1982

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INTRODUCTION

Problem solving has been considered, by mathematics teachers, to be of utmost importance and significance in their discipline. Problem solving has been found to be a unifying concept of all interdisciplinary skills which help achieve a common goal--an educated adult who can be an asset to society.

The roots of problem solving can be traced as far back as 300 A.D. Such well known mathematicians and philosophers as Pappas, Descartes, Euclid, Leibnitz, and Bolzano were concerned with the discipline of problem solving and the system of heuristics (Polya, 1965).

Numerous authors have discussed the need of transferable skills which, according to Piaget, begin on a concrete operational stage and progress to a formal operational stage (Piaget, 1972). One author in particular has expressed this need very succinctly:

> Problem solving is of ubiquitous importance whenever knowledge must be flexibly used (i.e., in any endeavor requiring more than the rote memorization or recall of information). Hence, problem solving plays a central role in all the sciences and in most other fields. Furthermore, there is an increasing need to teach improved problem solving skills to students who must be adequately prepared to cope with a world characterized by growing complexity, rapid change, and vastly expanding knowledge. (Tuma, 1980:ix)

STATEMENT OF THE PROBLEM

The whole meaning of problem solving in mathematics is putting mathematics to use. The ability to solve problems is basic not only to mathematics but to a wide variety of daily experiences. The use of mathematics should not be regarded as an afterthought to the development of concepts and skills. Rather, applications of mathematics must be a fundamental part of many lessons.

Students should be assisted in the plan of attack of a problem. Students should be encouraged to be creative and yet be logical in their thinking. They should try to understand the problem by sketching some figures, diagrams or tables summarizing the information given; determine a planning strategy on how to get started by breaking the problem into small increments and estimate the solution; carry out the plan of attack, reflecting occasionally on whether the approach is contributing to the solution; analyze whether the approach used seems to be the most logical and whether the use of the calculator would facilitate matters. The problem solving process also involves interdisciplinary investigations of problems that arise in everyday living.

PURPOSE OF THE STUDY

Mathematics is a logical system which exists as a human invention, formulated, enriched, extended, and revised in response to the twin needs of its development as an independent logical structure and its use as a convenient language for describing certain aspects of nature as seen by man. Mathematics permeates all facets of human life. Every individual needs an understanding of and proficiency in mathematics for personal living and for effective citizenship. Beyond this basic level, many jobs and professions require a more sophisticated use of mathematics than that required for daily living. Further, mathematics in its applications to scientific and technological developments impinges heavily if not always perceptibly on the lives of all. Developing problem solving skills will help students become an asset to society which is the purpose of education as a whole.

ORGANIZATION OF THE STUDY

The first section of this study examines the meaning of problem solving and its importance in the mathematical curriculum. Factors considered include the mental processes a student must go through in order to become a good problem solver, how problem solving is an interdisciplinary skill, and the importance of including problem solving in the mathematical curriculum. A curriculum should provide for a smooth transition from problem solving in the classroom to problem solving in the everyday life of an individual as an adult. This is needed in order to insure the achievement of one of the purposes of education which is to develop an individual who can function well as an asset to society.

The core of the study is devoted to examining the different strategies and techniques that could be followed by a teacher in a classroom in order to facilitate the thinking processes needed in a student. These procedures are not presented as foolproof methods but rather as approaches that may develop and achieve these skills in a student. Since the development of these skills is a priority of teachers, the major concern of this study is devoted to examining this aspect of problem solving as thoroughly as possible by using sources as far back as Polya, 1948, to the present time.

The last section of this study is devoted to the variety of applications of problem solving in the typical classroom. Various examples of these applications were examined in this study from kindergarten through high school.

The annotations were arranged in the following manner:

--in the first and third sections of this study, the annotations were arranged in alphabetical order since no particular pattern of presentation was thought necessary;

--whereas in the second section, the section devoted to strategies and techniques, the annotations were arranged in chronological order to insure more facility in comparing and contrasting the suggested methods recommended by authors throughout the years.

LIMITATIONS OF THE STUDY

The topic of problem solving has been of concern throughout the ages not only to mathematicians but to other academic and social fields as well. Therefore, the amount of material available on this topic was extensive and a thorough review of the topic in general was outside the scope of this study. The topic of problem solving as to how it related to the mathematical curriculum in regards to philosophy, strategies and techniques, and applications was analyzed in this study.

The time factor of this project was found to be a limitation as it was difficult to cover all aspects of problem solving in the above mentioned areas as well as would have been preferred. Readers who seek a more thorough examination of the topic of problem solving are referred to the National Council of Teachers of Mathematics (NCTM) Yearbook edited by Doyal Nelson, the NCTM 1980 Yearbook edited by Stephen Krulik, and the Educational Resources Information Center, ED 200 421.

Misleading titles of articles of various journals were found to be a limitation. It is suspected that several pertinent articles on the topic of problem solving may not have been selected due to this fact.

Lastly, since problem solving is a major concern at all grade levels of education, an overview of each general category was found to be more practical since a study at each grade level would have been too extensive to cover within the scope of this study.

GLOSSARY

audio-visual aids -- materials such as tape recordings, films, filmstrips, overhead transparencies, etc.

cognitive skills -- skills which rely on the repetition of facts or data.

concept of egocentrism -- theory of Piaget's that states that children's thinking is self centered in early childhood.

conceptual map -- visual representation of the interrelationship of various concepts in a particular discipline (See Appendix B).

concrete operational level -- a developmental level described by Piaget which requires concrete objects and a level of thinking where mental operations are object bound. This is usually between the ages of seven and eleven.

deduction -- reasoning from the general to the specifics.

domain -- the input to a particular situation.

flow chart -- a visual representation of a particular process (See Appendix A).

formal operational level -- a developmental level described by Piaget in children are able to reason verbally with reference to objects. This is usually from the age of eleven on.

generalizing -- a synthesis of two or more previously learned concepts.

heuristics -- a means and method of problem solving which involves an element of discovery.

hypothesis -- an unproved theory tentatively accepted to provide a basis for further investigation.

induction -- reasoning from specifics or particular facts to a general conclusion.

interdisciplinary -- relating to other academic disciplines in a particular school curriculum.

instructional variables -- the role of instruction in enhancing or retarding problem solving abilities or process.

LD -- Learning Disabled.

manipulative aids -- models which students can have some hands-on experience.

models -- concrete representations.

NAEP -- National Assessment of Educational Progress.

NCTM -- National Council of Teachers of Mathematics.

nontransformational problems -- problems in which no movement is implied. i.e., There are three frogs on a rock and two under the tree. How many frogs are there in all?

number phrase -- a mathematical expression.

number sentences -- mathematical equations or inequalities.

operations -- addition, subtraction, multiplication, and division.

paraphrasing -- restating in one's own words.

post-tests -- tests administered after a specific activity has been completed.

pre-tests -- tests administered before entering upon a specific activity.

primitive problems -- problems which can be easily solved with little intellectual effort.

problem -- a situation which involves personal involvement, blockage of achieving a goal exists, and thinking of a method of removing this block must occur.

process variables -- subject ability to select and organize appropriate strategies.

range -- the output to a particular situation.

SAT -- Scholastic Aptitute Tests

split-brain -- when the two hemisphers of the brain have been disconnected and each of the hemispheres work independently.

step problems -- problems which involve several steps in their solution.

subject variables -- characteristics of the problem solver which may be an asset or defect.

task variables -- factors contributing to the difficulty of the problem.

transferability -- potential applicability to a wide variety of particular problems but also the ability of an individual to use the skills.

transformational problems -- problems in which movement is implied. i.e., There are three frogs in a circle and two more frogs hop in. How many frogs are there altogether?

ANNOTATIONS

THE PHILOSOPHY OF PROBLEM SOLVING

Meaning of Problem Solving

Cohen, Louis S., and Johnson, David C., "Some Thoughts About Problem Solving," The Arithmetic Teacher 14:261-262, April, 1967.

The topics the authors dealt with were those which had been traditionally considered problem solving topics by elementary teachers and what was currently being considered as an essential addition to the teaching of problem solving in the elementary school. The meaning of problem solving was thoroughly discussed as was the role of word problems in the mathematical curriculum. Cohen and Johnson stated that good problems could require many of the following activities: observing, supplementing, exploring, decision making, organizing, recognizing, remembering, regrouping, isolating, combining, verifying, diagramming, guessing, classifying, formulating, generalizing, and applying. They thought that the most useful tool in good problem solving was the ability to translate a given situation into mathematic symbolism.

DeVault, M. Vere, "Doing Mathematics in Problem Solving," The Arithmetic Teacher 28:40-43, April, 1981.

DeVault proposed that as writing in the language arts, problem solving in mathematics required competence in certain basic skills or tools. However, so much time in the elementary schools is devoted to the perfection of these skills that little time is left for using these tools in actual problem solving. DeVault defined problem solving as occurring when an individual seeks to answer a question for which that individual has no readily available strategy for determining the answer. Therefore, if the strategy is already predetermined or known, no problem solving takes place. Four specific types of problems were discussed and charts were provided in which these four types of problems were analyzed. The remainder of the article was devoted to developing problem solving competence in students. According to DeVault, requirement for competence in problem solving was to be able to create descriptive and abstract models. The problem solving design proposed by the author was one that involved debate, discussion, and sometimes disagreement among the students when it was used in the classroom.
(DeVault is a professor of curriculum and instruction at the University of Wisconsin.)

Krulik, Stephen, "Problem Solving: Some Considerations," The Arithmetic Teacher 25:51-52, December, 1977.

Krulik gave three criteria which constituted identifying a problem: 1) personal involvement, 2) blockage of achieving an individual goal existed, and 3) thinking of methods of removing this block must occur. It was pointed out that there was no clear cut definition of a problem. The author merely gave the above criteria as a basis for identifying one. The acceptance of the learner was what was needed for a problem to exist not merely the statement of the problem in itself. Problem solving, according to Krulik, lied somewhere between total recognition and total confusion. Problem solving should be viewed as a process rather than a repetition of a solution. Students should analyze their methods of approach. Students should be encouraged to have an open minded approach to solving problems. Often times the inflexibility of the mind of the student causes the failure of thinking of a solution. The attitude toward problem solving was of utmost importance to Krulik. Also, six strategies were offered by the author which would lead to good problem solvers.
(Krulik is a professor of mathematics education at Temple University.)

Moursund, David G., Calculators in the Classroom: With Applications for Elementary and Middle School Teachers, John Wiley & Sons, Inc., New York, p.34-53, 1981.

According to Moursund, a problem is a situation in which there is a difference between how things are and how one would like them to be. There are numerous types of problems in our world - academic as well as social. Problem solving is a unifying theme in education. Some problem examples were given in expressing this thought. The definition of a problem consisted of three parts: given, goals, and restrictions. Definite problem situations were given and were shown to follow this definition. Problems were categorized into "primitives" and "step problems". The primitives were the basic problems easily solved with little intellectual effort. These were considered to be the building blocks for problem solving which involved several steps - the step problems. Five steps were provided for good problem solving. These steps were: understand, represent, plan, execute, and examine. Finally, Moursund believed that the calculator should be used in this entire problem solving process to facilitate computation.
(Moursund is a professor of mathematics at the University of Oregon.)

Robinson, Edith, "On the Uniqueness of Problems in Mathematics," The Arithmetic Teacher 25:22-26, November, 1977.

According to Robinson, problem solving did not pertain exclusively to the mathematics class but to science, social studies, and many other subject areas. The main purpose of this article was to explore the differences other than content between problems in mathematics and problems in other subject areas. Robinson stated that there were two basic types of verbal problems: exercises and problems requiring creativity or originality. These two types of problems were elaborated on in this article. A list of the characteristics of problems was offered for discussion and debate. Some of these characteristics were: problems that require reflective thinking, problems that require deduction, problems that involve transformations, problems that explore the domain of definition, and problems that emphasize relations in general. These characteristics were discussed thoroughly and several examples were provided for each one.
(Robinson is working with Educational Testing Service in Test Development.)

Skypek, Dora Helen, "Problem: Solving Problems," The Arithmetic Teacher 25:5, November, 1977.

Skypek considered problem solving to be the basic skill in mathematics. By problem solving was meant not only just knowing what to do but also a kind of commitment or willingness to tackle a problem when the solution to the problem was not easily attained. This article served as an introduction to the several problem solving articles in this particular issue of The Arithmetic Teacher.
(Skypek is a member of the National Council of Teachers of Mathematics (NCTM) and is a member of the editorial panel.)

Suydam, Marilyn, and others, "An Overview of Problem Solving in Elementary School Mathematics," pp. 101-108, R & D Speaks in Junior High School Mathematics: A Research Seminar for Practitioners, Educational Resources Information Center, ED 186 274, National Institute of Education, Washington D. C., September, 1979.

Suydam stressed that knowledge, skills, and understanding were important skills but in problem solving all these skills were synthesized for the purpose of achieving a goal, answering a question or reaching a decision. Four questions were addressed in this paper by Suydam. These four questions were: 1) What is the relationship of problem solving to basic skills? 2) Exactly what is meant by problem solving? 3) How should problem solving be taught? and 4) Whose responsibility is it to help students develop problem solving abilities? When should problem solving skills be introduced in the curriculum? A flow chart was provided which outlined a method of teaching problem solving by teachers in the classroom.

Implications of Problem Solving

Botts, Truman, "Problem Solving in Mathematics," Mathematics Teacher 58:496-500, October, 1965; 58:596-600, November, 1965.

Botts described the role of problem solving in the mathematical school curriculum. Of all the topics in mathematics, Botts believed that problem solving was of utmost importance. The bulk of these articles was devoted to illustrating this importance. Also, several problems were examined. Botts proposed that one good problem could lead to another one and yet another one. This process was believed to be ongoing.

Brulle, Andrew R., and Brulle, Christine G., "Basic Computational Facts: A Problem and a Procedure," The Arithmetic Teacher 29:34-36, March, 1982.

The authors stated that one basic fact that needed to be kept working on was the interdependence of group oriented contingency programs. Programs that stressed topics in successive form. They stressed that if this was true, then problem solving techniques needed to be introduced early in a school curriculum. Problem solving skills are needed in each of the curriculum areas. If these skills are introduced in the middle grades rather than earlier in the elementary grades, the students will be at a loss. The authors believed that the techniques needed for good problem solving in students should be introduced as early as possible in the school curriculum to facilitate the learning process of students in all areas of learning not only in mathematics.
(Andrew R. Brulle is an assistant professor in the Department of Special Education at Eastern Illinois University of Charleston, Illinois. Christine G. Brulle is a supervisor of student teachers at Eastern Illinois University of Charleston, Illinois.)

Burton, Grace M., "Word Problems-A Mirror of Society," The Arithmetic Teacher 21:33-36, January, 1974.

Burton pointed out that word problems reflected the aspects of the culture of the time, i.e., in 1871, travel problems referred to horsedrawn carts and trains whereas in 1971, travel referred to cars and jet planes. Therefore, the authors of textbooks incorporate word problems to reflect the age that we are living in. Not only was travel one of the visible cultural aspects used, but occupations were quite exemplary. This was done to catch the child's attention and make the word problems relevant to him/her. Therefore showing the importance of relevancy in problem solving to the reader. The reflection of real life situations in problem solving was very basic and essential in order to develop good problem solvers.
(Burton is a mathematics specialist at the Laboratory School at Eastern Connecticut State College.)

Krulik, Stephen, and Reys, Robert E., Problem Solving in School Mathematics, 1980 Yearbook of the National Council of Teachers of Mathematics, Reston, Virginia, 1980.

This NCTM Yearbook was totally devoted to the topic of problem solving. The yearbook consisted of twenty-two papers or essays ranging from the philosophy of problem solving, strategies, and useful applications. Krulik and Reys included essays which helped teachers of mathematics teach problem solving to students. The well known heuristics of George Polya were illustrated inside the front and back covers. Ideas which the teacher could use in the classroom were included. Krulik stated that problem solving should be the major focus of all mathematics instruction. One paper, on pages three through nine, dealt with problem solving as a goal, a process, and as a basic skill. This paper was written by Nicholas H. Branca. In this paper, Branca gave several examples which illustrated the implications of teaching mathematics using problem solving as a goal, a process, and as a basic skill.

Mertz, Donald W., Problem-Solving: An Overview of the Theory and a Course Attempting to Apply it, Center for Academic Development, Missouri University, St. Louis, Missouri, April, 1981.

Mertz noted that problem solving has been viewed with increasing interest throughout the years. Problem solving has been found to deal with cognitive skills which are applied to general problems. The particular cognitive skills needed to be taught to insure good problem solving abilities were discussed in this selection. Jean Piaget, Allen Newell, and Herbert Simon were noted by the author. Their cognitive psychology was analyzed. Piaget was analyzed in great detail. From this reading, certain teaching implications were drawn and a course entitled "Patterns of Problem Solving" taught under the Department of Philosophy at St. Louis University was examined.
(Mertz is a professor at the Missouri University, St. Louis - Center for Academic Development.)

O'Brien, Thomas C., and Shapiro, Bernard J., "Problem Solving and the Development of Cognitive Structure," The Arithmetic Teacher 15:11-15, January, 1969.

The authors stated that one basic purpose of teaching mathematics was to develop students' problem solving skills. They referred to Piaget's levels of development from activities which require concrete operation to formal or mental operation. The authors also gave reference to Piaget's concept of egocentrism which was essential to his theory of cognitive development in a child. The implications these thoughts have in a classroom situation were mentioned and examples of a classroom activity were discussed. One important result of this analysis was that social interaction in a child is essential to learning. Therefore, when working problems, the students should be given opportunities to work together.
(O'Brien is an assistant professor at Boston University.
Bernard is an assistant professor at Boston University.)

Quintero, Ana Helvia, Conceptual Understanding in Solving Multiplication Word Problems, Educational Resources Information Center, ED 201 497, Paper presented at the Annual Meeting of the American Educational Research Association, Los Angeles, California, April, 1981, 36 pp.

Quintero investigated the difficulties students have solving word problems using multiplication in their solutions. Some of the problem areas involved ratio, proportion, and factorization. Sixty children were involved and each child's performance was analyzed as he/she attempted different tasks. Some of the tasks were on the concrete level, that is, model forming, recognition, and construction or drawings. The errors these children made were analyzed in each case. The most difficult aspect of multiplication problems was found to be an understanding of what ratio really meant. This stressed the importance of conceptual understanding in solving word problems. Teachers need to have a better understanding of the conceptual structure of the domains that they are asking children to master. The remainder of the article was devoted to substantiating the relevance of this study. (Quintero is a professor in the department of mathematics at the University of Puerto Rico.)

Thornton, Carol A., and Bley, Nancy S., "Problem Solving: Help in the Right Direction for LD Students," The Arithmetic Teacher 29:26-41, February, 1982.

Thornton and Bley pointed out that problem solving skills have usually been reserved for middle school and upper elementary grades. They stressed the importance of beginning problem solving skills before these grades. Problem solving involves a great deal of decision making and this must be developed in the early grades rather than being delayed until junior high. Decision making, according to Thornton and Bley, was one of the most important aspects of problem solving and should be entered into even at the readiness level. The Learning Disabled (LD) child needs work to develop the needed skill of language, visual memory skills, and a high degree of reasoning, all of which will be difficult for them. Certain areas which were mentioned as needing reinforcement were listed and each was explained in detail. Also, suggestions were given for providing assistance in each troubled area.

Wheatley, Grayson H., "The Right Hemisphere's Role in Problem Solving," The Arithmetic Teacher 25:36-38, November, 1977.

Wheatley addressed the problem solving situation by studying the right and left hemisphers of the brain to see if this had any effect on problem solving abilities. The mental processes that contributed to our understanding or problem solving were thoroughly discussed. Recommendations for the teaching of problem solving were provided. The dual nature of our minds was stressed and the split-brain theory was stated. Wheatley stressed that the left hemisphere thinks in words whereas the right hemisphere thinks in images. The right hemisphere was more capable of synthetic and original thought. Therefore, this hemisphere played a more important role in the first phase of problem solving, determing the method to be used. Wheatley concluded that problem solving would be enhanced by greater use of the right hemisphere of the brain. (Wheatley is chairperson of mathematics and science education at Purdue University.)

Zur, Modecai, and Silverman, Frederick L., "Problem Solving for Teachers," The Arithmetic Teacher 28:48-53, October, 1980.

The authors believed that one common weakness of teacher training programs was the concentration on skills at the expense of thinking. Congitive mathematical behavior can be classified into three broad categories or levels: 1) memorization of facts, definitions or rules and procedures, 2) mental activity of generalizing or transferring knowledge from one learning context to another similar one, and 3) reorganizing and restructuring the variables in a problem so new relationships result. The third category was thought to be missing in the training sessions. The core of this article was devoted to developing a challenging mathematical thinking process for teachers. Several good examples were provided which would enhance thinking on the part of the students.
(Zur is dean of education at Bet Berl Teachers' College in Israel. Silverman is an assistant professor in the College of Education at Louisiana State University in Shreveport.)

THE STRATEGIES OR TECHNIQUES OF PROBLEM SOLVING

Recommendations before 1960

Polya, George, How To Solve It, Princeton University Press, Princeton, New Jersey, 1948, 225 pp.

This book could be considered a classic handbook of problem solving techniques, organized around four main themes: understanding the problem, devising a plan, carrying out the plan, and looking back. A "dictionary of Heuristics" was included. Polya referred to Pappus, a Greek mathematician, who reported about a branch of study which was called the art of solving problems in the seventh book of "Collectiones". Polya brought out that Pappus not only discussed the meaning of heuristics but also discussed its use. This was done by Polya to bring out the fact that the strategy of heuristics was not something new. This method had been used by famous men as Euclid, Apollonius of Perga, and Aristaeus the elder. Also, Polya pointed out that this heuristic reasoning which is recommended is based on induction. Several problems and hints to their solution were given.

Polya, George, How To Solve It, Doubleday & Company, Inc., Garden City, New York, 1957, 253 pp.

This book was a newer version of Polya's 1948 book with the same title. In addition to the above material, Polya stressed that if the majority of time in school was devoted to drill and routine operations, the interests of the students would be killed and their intellectual development would be hampered. Polya dealt a great deal with the requirements of students and teachers. Heuristics was one strategy that was stressed heavily by Polya. The mental operations students should be taught to go through which would lead, through common sense, to a solution of a problem were discussed. Again, Polya concluded with several problems and their inductive solutions.

Recommendations of the 1960's

Polya, George, Mathematical Discovery Volume II: On Understanding Learning and Teaching Problem Solving, John Wiley & Sons, Inc., New York, London, Sydney, 1965, 183 pp.

Polya made reference to famous mathematicians such as Descartes and Leibnitz. Descartes meditated upon a universal perfect method for solving all problems. Leibnitz very clearly formulated the idea of a perfect method. Useful patterns were expressed in this book which were most characteristic of human activity in problem solving. Part one of this book was devoted to familiarizing the reader to useful patterns. Part two outlined some steps to clarify the reader's mind and to improve the reader's problem solving ability. The strategy of heuristics was defined as a means and method or problem solving. Concrete examples were given throughout this book. Polya dealt with elementary problems as well as with high school problems. Techniques for approaching each of these problems were discussed. Some of the problems discussed were purely routine in nature while others required some degree of independence, judgment, originality, and creativity. (Polya is professor emeritus of mathematics at Stanford University.)

Johnson, David C., "Unusual Problem Solving," The Arithmetic Teacher 14:268-271, April, 1967.

The primary concern of Johnson was to exhibit a situation developing the problem solving ability of the reader. This problem which was stated was thoroughly discussed by Johnson. This problem did not lend itself to the immediate writing of a mathematical sentence nor to its solution. Rather, the basic objective was to solve the problem through discovery and generalizing from certain given data. Johnson argued that this was a unique method or technique which teachers should make use of in teaching problem solving skills. The remainder of the article was devoted to the sample problem. (Johnson is an assistant professor of education and chairperson of mathematics at the University of Minnesota High School, Minneapolis, Minnesota.)

Lyda, W. J., and Duncan, Frances M., "Quantitative Vocabulary and Problem Solving," The Arithmetic Teacher 14:289-291, April, 1967.

The authors limited their work to second grade students. The research on the quantitative vocabulary of second grade students was the topic of discussion. The design of the study was given as well as the number of testing materials and results. The conclusion and recommendation of the study was that in terms of the pupils, direct quantitative vocabulary contributed significantly to growth in problem solving. The authors recommended that teaching this type of vocabulary to students was a good technique. (Lyda is the graduate dean at Central State University in Wilberforce, Ohio. Duncan is a teacher at Peach County Training School in Fort Valley, Georgia.)

Sims, Jacqueline, "Improving Problem Solving Skills," The Arithmetic Teacher 16:17-20, January, 1969.

Sims gave several examples of interesting approaches to the improvement of problem solving skills in students. The examples pertained to reading, selecting, and evaluating material. Some of the suggestions referred to were: 1) using problems without numbers, 2) locating relevant and irrelevant data, 3) finding more than one question in a problem, 4) finding more than one way to work a problem, 5) use of diagrams, and 6) estimating a reasonable answer. Each of these suggestions was discussed thoroughly and examples of each were given.
(Sims is a resource teacher in the Fresno City Unified School District.)

Pottenger, Mary Joe, and Leth, Leonard, "Problem Solving," The Arithmetic Teacher 16:21-24, January, 1969.

The authors defined a problem as any situation where a student solved questions through something more than rote memory. They believed that the use of problem solving techniques should be encouraged in order to create an exciting atmosphere for learning in mathematics. Thinking for a purpose should be an aim of mathematics. They pointed out that an essential ingredient of problem solving was the desire, will, and resolution to solve problems. Also, the more senses a student involved in problem solving, the more involved a student would be. Furthermore, the role of the teacher was an important factor in the problem solving process. Three types of problem solving techniques were illustrated in this article and an example of each was given. The three types were: 1) algebraic type solutions using charts and record keeping, 2) concrete-activity or trial and error type, and 3) the nonverbal types which are solved without talking, merely through sketches and gestures or through facial expressions.
(Pottenger is a teacher at Homedale Junior High School in Idaho.
Leth is a teacher in Buhl High School in Idaho.)

Recommendations of the 1970's

Henney, Maribeth, "Improving Mathematical Verbal Problem Solving Ability Through Reading Instruction," The Arithmetic Teacher 18:223-229, April, 1971.

Henney argued that all methods and strategies were useless if students were not able to read the problem accurately. The reading of verbal problems requires more intense concentration than usual. Henney stressed that rereading is often necessary. Developing the ability to read verbal problems in students depends heavily on their vocabulary level. Tables were given in which the process from 1-step to 2-step problems was discussed in detail. Henney believed that the steps involved in analytical reading of verbal problems should be introduced gradually throughout the curriculum rather than in one section of the curriculum in the mathematics classrooms. Students should be able to read problems which have extra information which is not needed and be able to determine this. Also, students should be able to distinguish that there is missing information for the solution of a particular problem at hand. Therefore, Henney recommended stress on reading development as a technique in problem solving.

Higgins, Jon L., "A New Look at Heuristic Teaching," Mathematics Teacher 64:487-495, October, 1971.

Problem solving methods and techniques were discussed by Higgins. The technique recommended by Higgins was the heuristics approach to problem solving. It was believed that more student participation would be an effective approach to solving word problems. When students participated in the teaching as well as the learning process of problem solving, their personal involvement provided the motivation that was needed to be successful.

Crosswhite, F. Joe, and others, Teaching Mathematics: Psychological Foundations, Charles A. Jones Publishing Company, Worthington, Ohio, 1973, 260 pp.

The authors discussed not only the method of problem solving but also the meaningfulness of problem solving. The prerequisites needed to enable a student to be a problem solver were pointed out. Not only was the nature of mathematics' learners examined but also the nature of the learning processes in mathematics was studied by the authors. The authors recommended the heuristics approach to problem solving.

Richardson, Lloyd I., "The Role of Strategies for Teaching Pupils to Solve Verbal Problems," edited by Robert E. Reys, The Arithmetic Teacher 22:414-429, May, 1975.

According to Reys, problem solving is at the heart of mathematics. Reys stated that problem solving was a skill that continually needed development and refinement. Not only were students frustrated by word problems but teachers were also. The need for successful problem solving experiences was desperately needed in the lower grades. Richardson described two instructional strategies for good problem solving. These techniques could be learned and taught. Outlined in this article were two viable alternatives. Problem solving will be successful and more efficiently accomplished by students and with a greater confidence if teachers would follow the strategies mentioned herein. The strategies should be introduced in grades one through six if students are to be competent by the junior high grades, according to Richardson. The mastery of these skills would be seen in grades seven through ten. The results of Richardson's investigation and the implications for teaching were given at the end of this article.
(Richardson is a professor at the University of Missouri, St. Louis, Missouri. Reys is a professor at the University of Missouri, Columbia, Missouri.)

Davidson, James E., "The Language Experience Approach to Story Problems," The Arithmetic Teacher 25:28-35, October, 1977.

Davidson began his article by mentioning some of the reasons for students' failure with word problems. Among these reasons were: low reading comprehension, lack of experience with the problem situation, unfamiliarity with the language of the situation, and irrelevancy of the problem to name a few. The strategy Davidson proposed was the "Language Experience Approach". This method short-circuited the reasons for failure as mentioned above. This method merely had children write their own problems in their own language, using their own personal experiences. This eliminated the lack of readibility, lack of experience with the situation, and the unfamiliarity with the situation. Several sources for these problems were mentioned. One popular activity involved the mapping of the school grounds. Several other interesting examples were given. Problem solving, according to Davidson, was found to be enhanced by using this method and therefore we should not ignore it.
(Davidson is an associate professor of mathematics education in elementary education in Eastern Kentucky University.)

Lester, Frank K., Jr., "Ideas About Problem Solving: A Look at Some Psychological Research," The Arithmetic Teacher 25:12-14, November, 1977.

This article was written for the purpose of bringing out some of the valuable ideas that psychological research has provided on problem solving. Problem solving was considered to be a very complex process. Lester said that our knowledge of problem solving was more of a hodgepodge of information rather than a cohesive body of knowledge. The psychological research on problem solving is about as versatile as our own approach. Therefore, Lester recommended that each individual use his/her own technique to solving word problems.
(Lester is an assistant professor of mathematics education at Indiana University, currently on leave and at Kelvin Grove College in Queensland, Australia.)

LeBlanc, John F., "You can Teach Problem Solving," The Arithmetic Teacher 25:16-20, November, 1977.

Guidelines were given in this article which could be followed in teaching problem solving. The author stressed that these were not given as fool-proof guaranteed formulas for success. The special guidelines and hints were explored in detail. LeBlanc stressed that the instructional goal in teaching problem solving should be to help children learn procedures for problem solving. These procedures might be easy to state but are quite difficult to teach since they do not deal with specific content as in computational skills or in concept attainment. Four procedures or techniques were provided that LeBlanc considered inherent to good problem solving. These procedures were discussed and illustrated in this article. The illustrations were followed by some guidelines and instructional moves that could be used to help students. Also, a distinction was made between general strategies and helping strategies.
(LeBlanc is chairperson of the Mathematics Education Department in the School of Education and director of the Mathematics Education Development Center at Indiana University in Bloomington.)

Suydam, Marilyn N., and Weaver, Fred J., "Research on Problem Solving: Implications for Elementary School Classrooms," The Arithmetic Teacher 25:40-42, November, 1977.

The authors found that the heart of the problems or the characteristic of the difficulties with problem solving was the strategies that children used in the process. The process of problem solving itself was what Suydam and Weaver believed was of utmost importance. The main findings of several studies involved in problem solving were summarized in this article. Twelve specific techniques were itemized, discussed, and explained. Five common errors made by students were discussed. Finally, nine factors which may characterize those good at problem solving were listed.
(Suydam is a professor at the Ohio State University.
Weaver is a professor of the School of Education at the University of Wisconsin.)

West, Tommie A., "Rx for Verbal Problems: A Diagnostic-Prescriptive Approach," The Arithmetic Teacher 25:57-58, November, 1977.

West proposed that sitting back and giving reasons why children have difficulties with problem solving did no good because what we should be concerned with is how could we help them be better problem solvers. In order to accomplish this, West suggested three diagnostic questions: 1) Does the child comprehend the problem? 2) Can the child translate the data into a computational format?, and 3) Can the child carry out the computation involved in the solution of the problem? The core of the article was devoted to examining in detail each of these questions. Examples were given for each situation and they were analyzed in order to study the children's problem solving techniques and thinking processes. In order to help children with problem solving, West proposed that teachers needed to use a diagnostic prescriptive approach.
(West is an elementary mathematics resource teacher in Montgomery County, Maryland.)

Liedtke, Werner, and Vance, James, "Simulating Problem Solving and Classroom Settings," The Arithmetic Teacher 25:35-38, May, 1978.

The authors proposed that teachers could discover what kind of classroom settings were condusive to problem solving attitudes through simulations. Several appropriate settings were described in this article. Four characteristics of good problems were given and the results of the children's involvement were readily seen. Three categories for good problems were stated and defined. The authors stated that the use of manipulative aids was found to be quite useful in the problem solving process. They also stressed that many problem solving situations and settings could be created without the use of expensive manipulative aids. Throughout the article, the authors pointed out that through the activities mentioned, the students would learn to appreciate the value of graphs as a skill and strategy in solving problems.
(Liedtke and Vance are both associate professors at the University of Victoria, British Columbia.)

Masse, Marie, "More Problems, Please," The Arithmetic Teacher 26:11-14, December, 1978.

Masse said that children would be asking, begging, for more problems if we, as teachers, would have developed problems that were short on computation and long on logical thinking. The main thrust of problem solving should not be on computational competence but rather on the development of logical thinking. The suggested manner in which this can be done is to take the "numbers" out of the problems. Therefore, the objective of such problems would no longer be to get the answer but rather the development of problem solving strategies. Discussion in the classroom should be on how the problems could be wolved. Specific steps, which could be taken to help solve these problems, were detailed in this article. In order to challenge the brighter students, the teacher might suggest that they make up similar problems. The remainder of this article was devoted to the listing of eight problems which were diagrammed and handled in this non-numbered fashion.
(Masse is a mathematics specialist at the Page School in West Newbury, Massachusetts.)

Conquering Word Problems in Mathematics, Glen Ellyn Math House, 1979.

This particular resource was a combination of cassettes and worksheets which gave practical ways of teaching problem solving. Several topics were covered from the meaning of a problem, the proper reading of a word problem, analyzing the proper procedure to be used in the solution of the problem to the checking of the answer. Approaching problem solving through audio-visual materials was considered to be a unique technique in developing problem solving skills in children.

Goldin, Gerald A., and McClintock, C. Edwin, Task Variables in Mathematical Problem Solving, Educational Resources Information Center, ED 164 328, National Science Foundation, Columbus, Ohio, 1979.

This document was a framework for research in problem solving. This framework defined variables which described problem solving tasks and placed them into various categories. The authors also gave very specific strategies and techniques for good problem solving which were described thoroughly.

Zweng, Marilyn J., "The Problem of Solving Story Problems," The Arithmetic Teacher 27:2-3, September, 1979.

Zweng argued that even though reading ability and computational ability were big factors in solving word problems, they were merely excuses and not reasons for poor problem solving abilities. These were not the major problems of students who had difficulty with word problems. The major stumbling block was found to be that of deciding which operation-addition, subtraction, multiplication, or division-should be used in the solution. The author found that textbooks gave little guidance on how to select the appropriate operation. Listing the given, to find, and answer of the problems was actually detrimental since it isolated the facts from the data that affected the problem. Estimating was also found not to be the answer since a child would have to know the operation involved before he/she could estimate properly. The answer provided by Zweng was to develop a language for the students which would enhance problem solving skills. That is, to have the students associate certain operations with certain words. Also, in order to be proficient, much practice is needed in problem solving. Besides offering this technique, Zweng suggested that a minimum number of six problems per week be provided for students to solve.
(Zweng is a professor of education and mathematics at the University of Iowa City, Iowa.)

Wheatley, Charlotte L., "Mathematics Experiments: An Alternative Teaching Strategy," The Arithmetic Teacher 27:18-21, October, 1979.

Wheatley simply described and discussed an alternative to the use of mathematics textbooks and their specific problems to teach mathematics and good problem solving skills. The alternative teaching strategy was a "math experient". A mathematical experiment was defined as:

> A math experiment is a small-group activity in which a problem is posed that can best be solved by collecting data, forming and testing hypotheses, and verifying results. When a problem is posed, children must decide how to approach the problem, what data to collect, and how to interpret the results. Finally, they must verify the solution.

One of the most important reasons for using the math experiment strategy, according to Wheatley, was the great amount of learning of problem solving techniques that resulted. The remainder of the article was devoted to the implementation of the math experiment in the classroom.
(Wheatley is an assistant professor in the department of education at Purdue University.)

Gathany, Timothy A., "Involving Students in Problem Solving," The Arithmetic Teacher 72:617-672, November, 1979.

One effective technique in promoting interest in problem solving, discussed by Gathany, was to give students opportunities for developing and solving their own problems. This personal involvement on their part would insure motivation. For each problem they wrote, the level of mathematics was not as complicated as textbooks would recommend. However, the setting would develop useful problem solving skills. This article was devoted largely to the exploration of some ideas that students could use in generating their own problems. Also, suggestions for implementing this type of activity and strategy in the classroom were made by Gathany.

Lindvall, Mauritz C., and Ibarra, Cheryl Gibbons, The Development of Problem-Solving Capabilities in Kindergarten and First-Grade Children, Educational Resources Information Center, ED 201 524, Learning Research and Development Center, Pittsburgh, Pennsylvania, 1979, 21 pp.

The authors investigated the strategies used by first grade and kindergarten students in solving simple arithmetic story problems successfully. The procedure used by these students had three stages: 1) comprehending the story, 2) building a general conceptual model, and 3) generating a mathematical model. The use of manipulative aids was found to be essential. This study was performed on eighty-one students and the results appeared to support the theory that the students who could successfully solve a word problem for the correct answer to the question being asked were also able to develop a general model of the problem itself. Concrete representations were needed. These children had little concepts of addition, subtraction, multiplication, and division. The authors analyzed their thinking process in each of the problems. The four stages the students went through were discussed also.

Meiring, Steven, "Problem Solving: A Basic Mathematics Skill," pp. 96-101, R & D Speaks in Junior High School Mathematics: A Research Seminar for Practitioners, Educational Resources Information Center ED 186 274, National Institute of Education, Washington D. C., September, 1979.

Meriing believed that one of the primary objectives for problem solving during the intermediate and junior high grades should be to provode a variety of problem solving situations for study from the text or from real-life situations. Some of the situations which should be emphasized included problems with superfluous or insufficient data to problem solving with more than one method of solution. This publication was a two volume publication which was designed to provide units for in-service study covering such topics as types of strategies and classroom management for problem solving plus sets of problems for students in grades three through nine. A portion of this publication was devoted to giving practice to teachers in applying the strategies mentioned by Meiring to problems provided in texts.

Suydam, Marilyn, and others, "Problem Solving in the Classroom," pp. 109-115, R & D Speaks in Junior High School Mathematics: A Research Seminar for Practitioners, Educational Resources Information Center ED 186 274, National Institute of Education, Washington D. C., September, 1979.

This paper was written by Suydam to discuss the techniques of grouping students, modes of presentation, external and internal variables, realistic modeling, and avoiding pitfalls by teachers in good problem solving situations. To improve one's effectiveness in problem solving teaching, many components effect the success and richness of such learning experiences. The exact meaning of the techniques mentioned above were given in this paper. A few of the pitfalls which should be avoided by teachers were: overemphasis on verbal clues, the suggestion of one best method, and the concept that problem solving is just for the better students.

Recommendations of the 1980's

Krulik, Stephen, and Rudnick, Jesse A., Problem Solving: A Handbook for Teachers, Allyn and Bacon, Inc., Boston, Massachusetts, 1980, 227 pp.

This handbook was primarily a handbook with scores of activities, problems, games, and blackline masters for teaching the skills and subskills needed for successful problem solving. Applications to the various grade levels were given. The illustrations given in this handbook were very appropriate. The strategies and techniques given herein were quite numerous and applied directly to the situations at hand. A unique flowchart of the problem solving process was presented and can be viewed in Appendix A of this project.

Izumo, P., Option X, Level B Problem Solving, Educational Resources Information Center, ED 201 472, Hawaii State Department of Education, Honolulu, Office of Instructional Services, March, 1980, 58 pp.

This guide outlined a course on problem solving techniques. The specific techniques discussed were: working with diagrams; organizing information; using patterns; simplifying problems, and looking back. Each of these was analyzed and discussed with several strategies mentioned on how to develop each objective. This document was a guide for the course for grades nine through twelve on problem solving. The different examples given were to stress the applications to real-life situations such as business, consumerism, and industry. The author also gave the teachers a guideline that would help develop good problem solving techniques in students.

Krulik, Stephen, and Reys, Robert E., Problem Solving in School Mathematics, pp. 136-145, 1980 Yearbook of the National Council of Teachers of Mathematics, Reston, Virginia, 1980.

Five problem solving strategies were presented by the authors of these selected pages. These five strategies were: trial and error; patterns; solving a simpler problem; working backwards, and simulation. Each of these techniques were discussed thoroughly and several examples of each were illustrated. Also, the inter-relationship of problems in mathematics and other disciplines was presented in detail.

Pikaart, Len, and others, 1980 Ohio Regional Conferences on Mathematics Education, Educational Resources Information Center, ED 200 421, National Science Foundation, Washington D. C., 1980, 287 pp.

In 1980, six conferences were held to provide teachers and any decision makers with problem solving and calculator experiences problem resources, strategies, guidelines, and summaries of research results. Focus was on junior high and secondary schools. Two of the purposes of these conferences were: 1) to help students develop and improve problem solving abilities, and 2) to provide resources and describe techniques for problem solving in the classroom. The major part of this document was compiled for participants and distributed to them. The hand-outs were to be resource materials for the participants. The various topics covered which pertained to problem solving were: assessment data; implications for problem solving; basic heuristics for problem solving; problem solving resources; posing and reposing problems; real-word problems, and the teaching of problem solving. Each of these topics was discussed extensively. Some suggested strategies were: look for a pattern; construct a table; consider possibilities systematically; act it out; make a model; guess and check; work backwards; make a drawing, figure, or graph; select appropriate notation; restate the problem (in your own words); identity wanted (gives needed information); write an open sentence; solve a simpler problem; change your point of view; identify a subgoal; check for hidden assumptions; use a resource; generalize; check the solution, and study the solution process.

O'Loughlin, Michael, and Fleischner, Jeannette, Story Problem Solving: Implications of Research for Teaching Children With Learning Disabilities, Educational Resources Information Center, ED 210 841, Office of Special Education, Washington, D. C., Columbia University of New York, 1980, 31 pp.

The authors reviewed both American and Soviet writings to present some of the essential parts of problem solving techniques needed by the learning disabled (LD) student. They discussed the necessary analytic and intuitive strategies. The LD child needs a specific method of going from concrete problems to symbolic problems. There are specific teaching methods which should be used in dealing with the LD students. Also, the authors found that the LD child often can present the solution to a problem orally but may not be able to do so in writing. Certain task variables, subject variables, process variables, and instructional variables were mentioned in detail. Finally, the findings of the authors were evaluated to assess their contributions toward a refinement of the methodology employed in the study of problem solving.

Heck, Shirley F., "Problem Solving Plus Role Playing Equal Interest Plus Skills in Mathematics," The Arithmetic Teacher 28:34-38, September, 1980.

Heck believed that motivation was the key in learning problem solving for students. Teachers should relate what the child is learning to his/her fulfillment of a real need or purpose in life. The Kernal Design (the gist or core of an idea) was described in detail. Heck stressed that this not only developed good mathematical problem solving motivation but also enhanced the idea of interdisciplinary learning.
(Heck is an associate professor of early and middle childhood at the Ohio State University.)

Burton, Leone, "The Teaching of Mathematics to Young Children Using a Problem Solving Approach," Educational Studies in Mathematics 11:43-58, November, 1980.

The purpose of this article was to report upon the early stages of a research project in London. This project was entitled "The Skill and Procedures of Mathematical Problem Solving in Pupils of 9-13 years". The study explored the need of such a project. The two main tasks that were attempted by this project were: 1) to serve as an inventory of problem solving skills, and 2) the designing and testing of a structured program through which 9-13 year olds could learn problem solving techniques. Freedom of discussion was found to be an essential component. Helpful guidelines were also presented in this article to promote good problem solving in student. A diagrammatic representation of problem solving was given in this article.

Yeotis, Catherine, and Hosticka, Alice, "Promoting the Transition to Formal Thought through the Development of Problem Solving Skills in Middle School Mathematics and Science Curriculum," School Science and Mathematics 80:557-562, November, 1980.

The authors proposed that problem solving was a reflective thinking process. John Dewey was quoted as a reference for this belief. The study of why students were not able to think at higher levels in college led to examining the intellectual thought processes of students at lower levels. Piagetian's models were used. The authors stated that the middle school level seemed like an ideal place to begin intensive study of problem solving for two reasons: 1) students are or are entering into the transitional period between concrete operational thought and formal operational thought, and 2) in the middle schools, mathematics and science are required subjects and thereby these processes may encourage students to take additional courses in high school. Each of these two were discussed and analyzed in detail. Also, a model for teaching problem solving skills was given where the teacher needed to go through three phases of instruction: 1) cue attendance, 2) step verbalization, and 3) step diagramming. The authors stressed that problem solving skills provide a vehicle for teaching critical thinking skills.

Fennell, Francis M., "The Need for Problem-Solving Skills," Elementary Mathematics: Priorities for the 1980s, pp. 21-22, Fastback 157, Phi Delta Kappa Educational Foundation, Bloomington, Indiana, 1981, 28 pp.

The National Assessment of Educational Progress (NAEP) report in September of 1979 showed that basically while computational skills in students were satisfactory, the ability of students to solve problems had declined during the 1973-1978 period. These skills were found to be lacking even in the most basic problem solving areas. This raised the concern of educators because the scores on the Scholastic Aptitude Test (SAT) had declined also. Fennell believed the ability to apply mathematics to solve problems was at least as fundamental as computational skills. Fennell stated that the basic purpose of learning mathematics was to be able to solve problems. A list of the types of problems students might encounter were given. Not only did Fennell state that instructural techniques were sorely needed by teachers, but some techniques were provided. Fennell provided a problem solving staircase or heuristics for the reader at the end of this section.
(Fennell is an assistant professor and coordinator of elementary education at Western Maryland College in Westminster, Maryland.)

Muller, Adelyn C., and Kurtz, Ray, "Students Like Personalized Word Problems," The Arithmetic Teacher 28:13-14, May, 1981.

The authors contended that no book could contain problems that relate to real problems in the experiences of all children. Each is such an individual that it is difficult to meet this challenge. On the other hand, all computation problems are all the same. Our unrealistic approach to word problems could be the cause of students having such a difficult time with them. The main purpose of the authors was to describe a way to develop personalized word problems. They found that ownership of a problem carried a high motivational impact. Several examples of this process were detailed in the remainder of the article.
(Muller was a mathematics consultant with the Shawnee Mission Schools in Kansas until her recent retirement.
Kurtz is a professor of curriculum and instruction in College of Education at Kansas State University in Manhatten, Kansas.)

Higginson, William, "Mathematizing Frogs: Heuristics, Proof and Generalization in the Context of a Recreational Problem," Mathematics Teacher 74:505-515, October, 1981.

Migginson's major belief was the fact that one phenomena that impeded problem solving skills was "functional fixedness". This fixedness causes a person to see a particular object or problem solely in terms of one of its features rather than having a more global view. The author's mathematizing of a problem was expressed as looking at a problem and seeing the way in which many various mathematical concepts are found or involved in it. Many times students cannot see the solution to a problem because of a limitation to one mathematical concept or theorem. In reality, problems have a number of features. These problems are not one-dimensional. By extracting as many concepts as possible from a problem, a better idea of how mathematics knowledge evolves can be seen. Mathematics is never finished. The remainder of the article was devoted to the development of heuristics and its application to specific problems. One final point was made by Higginson and that was that good problem solvers learned from their mistakes.
(Higginson is a professor at the Queen's University at Kingston, Kingston, Ontario.)

Hills, George L. C., "Through A Glass Darkly: A Naturalistic Study of Students' Understanding of Mathematical Word Problems," Instructional Science 10:67-93, October, 1981.

Hill presented a typical word problem and stressed that we really know little as to why students have difficulties with such problems. Perhaps what we should do is study what students actually do when confronted with problems of this type rather than looking at mere test scores. This study done by Hills was focused on a student's problem solving strategies and these strategies were compared with the strategies normally presented in a classroom. The results were that even though there are systematic and fundamental differences between the procedures prescribed and those actually used by the student, this was undetectible in the final answer. Therefore, Hills believed that more attention should be given to what students say and do if we are to understand why they are having difficulties with problem solving.

Jensen, Rosalie, and O'Neil, David R., "Let's Do It; Classical Problems for All Ages," The Arithmetic Teacher 29:8-12, January, 1982.

Children in middle schools need concrete and pictoral aids as well as guidance from adults in choosing strategies in the working of word problems. Real-life problems were found to be of great interest to the students. Focus should be on strategies rather than performance of complex arithmetic operations. Three such examples of this were found in this article. Three things should be pointed out to students: 1) several strategies may need to be tried before a suitable one is found, 2) there may be more than one solution to a given problem, and 3) sometimes one must back away from a problem for a while in order to get a fresh look before proceeding. The authors concluded their article by presenting a strategy for problem solving they believed to be effective.
(Jensen and O'Neill are professors at the George State University, Atlanta, Georgia.)

Krulik, Stephen, and Rudnick, Jesse A., "Teaching Problem Solving to Preservice Teachers," The Arithmetic Teacher 29:42-45, February, 1982.

Teachers need to be very specific in their terminology in the classroom. This was an opinion expressed by the authors of this article. There are specific differences among the terminology of question, exercise, and problem. They are not to be used interchangeably. Krulik and Rudnick said that teachers needed to become problem solvers themselves before they approached this aspect of learning with their students. They stated that for most preservice teachers, selecting a strategy was the most difficult and important part of the problem solving approach. A flow chart of the problem solving process was provided along with a list of strategies for good problem solving. The authors stressed that the best method of teaching problem solving was for the teachers themselves to be good models. A list of ten suggestions for teaching problem solving was given. Finally, five specific steps were outlined and elaborated on as to what teachers could do to become good problem solvers and good classroom teachers of problem solving.

Burns, Marilyn, "How to Teach Problem Solving," The Arithmetic Teacher 29:46-49, February, 1982.

Burns discussed ways of incorporating problem solving as an integral part of the mathematics curriculum in the classrooms. A typical classroom situation was discussed in regards to the amount of time which was devoted to computational skills which in the future would be eased by the use of calculators in everyday life. Why then spend so much of the classroom time on computation rather than on something more pertinent such as problem solving! In real life situations, one is rarely given all the information one needs in one tidy package. Therefore, students need to know how to collect data and not just decide what operation is needed to get the answer. Burns stated that problem solving should be a long range goal not a lesson objective. Children must have daily problem solving experiences where the important focus is on the thinking process involved and not on the speed of getting the answer.
(Burns is the author of several different children's books and conducts workshops in mathematics teaching for elementary and junior high teachers in various parts of the country.)

Duea, Joan, and Ockenga, Earl, "Classroom Problem Solving with Calculators," The Arithmetic Teacher 29:50-51, February, 1982.

Duea and Ockenga stressed the importance of making problem solving the focus of the school mathematics. Problem solving has been aided by the use of calculators in the classroom. They pointed out that the students themselves should be encouraged to be authors, to create problems as well as to solve them. The use of calculators aided students that may have some learning disabilities in the solution of problems. Students should take ownership of a problem and thereby develop a greater desire to solve them. Problem solving developed creative thinking. The authors believed that this kind of thinking was needed by students to become an asset to society. (Joan Duea is a third grade teacher at the Price Laboratory School and a professor of education at the University of Northern Iowa in Cedar Falls. Earl Ockenga is an assistant professor at the Price Laboratory School.)

LeBlanc, John F., "Teaching Textbook Story Problems," The Arithmetic Teacher 29:52-55, February, 1982.

Story problems have not only been a part of the mathematics curriculum for years but will continue to be a part of the curriculum for many years to come. LeBlanc pointed out that the question was not whether or not problem solving should be a part of the curriculum but rather that the difficulty of problem solving should be eased for students. The author proposed that problem solving skills could be taught. The problem solving process suggested followed the process of George Polya. Polya's process was discussed in great detail. The remainder of the article was devoted to relating the steps of Polya to a classroom situation by translating Polya's terms into classroom vocabulary. The framework suggested was one of tell, show, solve, and check. What problem solving involved was not what problems should be introduced but what problem solving process should be taught. (LeBlanc is a professor and chairman of mathematics education at the Indiana University in Bloomington, Indiana.)

Suydam, Marilyn N., "Update on Research on Problem Solving: Implications for Classroom Teaching," The Arithmetic Teacher 29:56-61, February, 1982.

Suydam mentioned that problem solving had been recognized as an important part of the mathematics curriculum. Several sources of research were mentioned in this article. Specific items and aspects of what is known to be true about problem solving were itemized and discussed in detail. What teachers could do to help students learn problem solving skills were mentioned. Among the strategies listed in this article were: modeling; systematic guided discovery; heuristics, and rephrasing. Six specific implications affirmed by research were not only mentioned but also discussed thoroughly by Suydam. The author's concluding comment was that when specific strategies were taught the aspect of flexibility must be included at all times in order to be effective. (Suydam is an adjunct professor of mathematics education at the Ohio State University.)

Tobin, Alexander, "Scope and Sequence for a Problem-Solving Curriculum," The Arithmetic Teacher 29:62-66, February, 1982.

The main theme of this article was that the performance in problem solving by our students which are our future will measure the effectiveness of our personal and national possession of mathematical competency rather than computational skills in themselves. Tobin believed that the mathematical curriculum should be organized around problem solving. The main reason that problem solving was made a separate strand in the mathematics curriculum was that this would enable teachers to recognize between verbal exercises and actual real problems. Handled separately it would be given the attention it so well deserved. Each level of the mathematics curriculum was addressed from level one to level eighteen. Several strategies for teaching problem solving for each of these levels were given and discussed by the author. (Tobin is a director of mathematics education for the school district of Philadelphia.)

Warman, Michele, "Fun With Logical Reasoning," The Arithmetic Teacher 29:26-30, May, 1982.

Warman stressed the importance of the ability of students to see patterns and relationships not only for academic success but also for personal success. These processes should be developed in the mathematics curriculum in the problem solving skills. Several objectives were stated and explained regarding the development of reasoning skills which are essential for good problem solving. Sequences and analogies were discussed. "Think" problems were introduced by Warman. Several examples of each of these were given and analyzed. A third group of problems were studied. This group of problems consisted of "critical reading" problems. The use of quantifiers basic logical arguments, and Venn diagrams were mentioned as helpers of word problem solutions. These techniques were believed to be effective methods of approaching problem solving. The remainder of the article was devoted to several exercises which illustrated each of the group of problems mentioned above. A summary was provided at the end of the article which enabled the reader to see how each of these problems were related.

Moser, James M., and Carpenter, Thomas P., "Young Children Are Good Problem Solvers," The Arithmetic Teacher 30:24-25, November, 1982.

The authors proposed that young children did not get the credit they deserved for being good problem solvers. they studied a group of students from grades one through three and found that they approach problems using strategies based on counting even before formal addition, subtraction, multiplication, and division are introduced. The students were able to make rational choices. They were also persistent and creative in their work. The students' individual strategies were pointed out and discussed thoroughly. Finally, the authors argued that not all children were found to be good problem solvers but the absence of total failure was encouraging.
(Moser is a senior research scientist and adjunct professor of mathematics education at the University of Wisconsin.
Carpenter is a professor in the Department of Curriculum at the University of Wisconsin.)

THE APPLICATIONS OF PROBLEM SOLVING

Applications in the Elementary Grades

Block, Karen K., and others, A Computer Resource for the Elementary School. Progress Report 1971-1972, Educational Resources Information Center, ED 076 304, National Institute of Education, Washington, D. C.; National Science Foundation, Washington, D. C., 1972, 48 pp.

Block dealt with many aspects of mathematics which could be facilitated with the use of computers. The portion which dealt with problem solving skills in an individualized classroom was entitled: Computer-Assisted Instruction in Problem Solving Techniques, by Richard A. Roman, pp. 31-34. In this section, Roman believed that the components of problem solving behavior emphasized in a classroom were of utmost importance. Three of these components were: 1) subdividing large problems into smaller ones, solving these and then integrating these solutions into a total solution; 2) formulating and testing hypotheses, and 3) deciding to continue working on a given path to a solution or changing to a new method. Roman stressed that teachers should concentrate on the student's own thought process. Also, Roman described a program in which this method was used and described the steps in achieving the specific goals and the specific computer instruction needed.

Carpenter, Thomas P., and others, "Solving Verbal Problems: Results from National Assessment," The Arithmetic Teacher 28:8-14, September, 1981.

In this article, the falsehood of reading ability as a source of problem solving difficulties was brought out because the problems to be solved were taped on an audio recorder and the children still had problems in their solution. The lack of computational skills did have an effect on the solution of the problems. The operations of addition, subtraction, multiplication, and division were carefully discussed in relationship to the solution of the problems. The mastery of these skills did have an effect on the solution of the problems. The nonroutine problems gave the students the greatest difficulty especially in the operation of multi-step problems. The solution of one-step problems was found to be greatly effected by the mastery of the particular operation. Although, the nonroutine problem was greatly effected because of the several operations involved. Teachers need to see that there is a transition from one-step problems to multi-step problems in their curriculum. The authors found that students from kindergarten through first grade actually could think a problem through and analyze it. Although, they found that young students preferred to use addition in preference to multiplication.

Goodman, Terry A., "Brian's Question," The Arithmetic Teacher 29:14, March, 1982.

Goodman stated that a good problem solving atmosphere in the classroom often gave way to good problem posing on the part of the students. A good example of a young boy about eight years old posing a problem from watching a television program was given. This problem involved a detective show in which the heroine ran off with one million dollars in a briefcase. The heroine was able to jump over fences and across ditches with the briefcase in her hand. The young boy asked if this was feasible - in relation to the weight of the briefcase. The result was that a project was set up in the class to discuss this problem. The students found that the briefcase would be too heavy to carry unless the bills involved were of a large denomination. Some points that were illustrated were that good problems involve decision making. Also, it was found that good problems often lead to other problems which are generated by students. A good problem solving atmosphere can encourage both problem solving and problem posing.
(Grossman is an assistant professor in the department of mathematics at Central Missouri State University in Warrensburg, Missouri.)

Grossman, Rose, "Problem Solving Activities Observed in British Primary Schools," The Arithmetic Teacher 15:34-38, January, 1969.

This article was a summarization of a visit to England which was devoted to analyzing the innovative primary school mathematics programs in the art of problem solving by visiting various schools. Some of the items which were observed and studied carefully were the use of physical materials, guidebooks, and activity cards. In addition to these, the role of the teacher, the role of the head teacher (equivalent to the principal of our elementary schools), and collaboration that took place between the teachers were analyzed. Many other innovative ideas were brought out in this article by Grossman.
(Grossman is an instructor in mathematics and mathematical education at Stern College, Yeshiva University.)

Jacobson, Ruth S., "A Structured Method for Arithmetic Problem Solving in Special Education," The Arithmetic Teacher 16:25-27, January, 1969.

Jacobson presented a lesson in problem solving to fourth graders with the objective of developing the children's ability to think about the word problems in terms of action, operation, and estimation. This was brought on by the realization of the author that problem solving was found to be a weakness of grades three and four. Specific suggestions of introducing this topic to the special education students were given.
(Jacobson works in the Department of Pupil Personnel Services at the Brookline Public Schools. Jacobson also worked in Union, New Jersey as a LD specialist.)

Liedtke, Werner, "The Young Child as a Problem Solver," The Arithmetic Teacher 24:333-338, April, 1977.

The younger child's approach to a problem situation was very different than the adult's approach. For a young child, the possibilities of no solution, no available proof, did not exist. Liedtke proposed that they rarely showed signs of frustration when they were involved in problem solving, no matter how long or difficult a problem was. When they were faced with a problem, a majority of young children were willing to try to solve it. Young children relied heavily on perception. Therefore, visual aids were important. The great majority of the article was devoted to giving of examples of problem situations and how young children analyzed their way to solutions. After each example, the young child involved was asked to tell why and how he/she knew the solution was correct. The young children were very specific in their answers.
(Liedtke is an associate professor at University of Victoria in British Columbia.)

Madell, Robert L., "Children Can Understand Mathematics," The Arithmetic Teacher 29:18-21, January, 1982.

The essence of this article was that one of the underlying ideas of the mathematics program should be to benefit children from some considerable period of time being devoted to solving problems in very elementary ways. If children were allowed to be exposed to problem solving situations, they would come to more direct instruction later with positive attitudes and good understanding. Madell's examples involved numerals, fractions, and concrete numbers. Examples were given on how problems could be solved physically through the operations of addition, subtraction, multiplication, and division. Two important points that Madell wanted to bring out were: 1) the form in which problems are presented to students, and 2) the pace at which students are introduced to problems.
(Madell is a mathematics specialist at the Village Community School at New York City.)

Nelson, Doyal, and Kirkpatrick, Joan, "Problem Solving," Mathematical Learning in Early Childhood, pp. 69-93, Thirty-seventh Yearbook of the National Council of Teachers of Mathematics, Reston, Virginia, 1975.

The central purpose of any instruction in mathematics at the early childhood level was to help a child see the relationship between day-to-day events and their mathematical structure. The authors proposed that problems should come from real situations. The characteristics of good problems were outlined thoroughly. Several examples and illustrations of young children using manipulative aids were included. Specific problems for beginning and later stages of problem solving were detailed. Several illustrations and photos of children involved in small group problem solving were included. In the concluding remarks, the authors again summarized the characteristics of good problems and gave suggestions for selecting and designing appropriate problems.

Steffe, Leslie P., "The Relationship of Conservation of Numerousness to Problem Solving Abilities of First Grade Children," The Arithmetic Teacher 15:47-52, January, 1968.

Steffe defined conservation of numerousness as the ability to match two sets, one to one, regardless of the arrangement of the two sets. This study involved a random selection of 132 first grade students. These students were given three tests: 1) a test of conservation of numerousness, 2) a test of eighteen problems of addition, and 3) an addition facts test. The results found that those given accompanying aids did better than those who were not able to use manipulative aids in their problem solving activities. Therefore, the author recommended the use of manipulative aids in teaching problem solving to young children.
(Steffe is a professor at the University of Georgia, Athens, Georgia.)

Szetela, Walter, "Analogy and Problem Solving: A Tool for Helping Children," The Arithmetic Teacher 27:18-22, March, 1980.

The main thrust of this article was to give teachers the suggestions that they needed to follow up hands-on experiences in determining area and volume with problem situations that were not only interesting but also led students to focus on the relevant attributes of the problem at hand. Four such problems were described and discussed thoroughly by Szetela. All four problems were compared and contrasted at the end of the article. Szetela stated that problem solving had been regarded as a neglected element in elementary mathematics. And, that analogy, in particular, was an effective way of exposing children to problem solving experiences. Szetela stated that this method was found to be effective with seventh and eighth grade students. The elementary grades found this method effective also.
(Szetela is an assistant professor at the University of British Columbia.)

Trueblood, Cecil R., "Promoting Problem Solving Skills Through Nonverbal Problems," The Arithmetic Teacher 16:7-9, January, 1969.

Trueblood believed that nonverbal problems eliminated the interference of low reading ability to the development of problem solving skills. These nonverbal problems could be called functional problems or experience units and should be introduced through or illustrated through sketches on the blackboard. Four advantages of using this type of problem solving in the elementary grades were given by Trueblood. Several good examples of each advantage were offered at the end of this article.
(Trueblood is a professor at Pennsylvania State University.)

Underhill, Robert G., "Teaching Word Problems To First Graders," The Arithmetic Teacher 25:54-56, November, 1977.

Five studies of addition and subtraction word problems were presented in this article. The results of these studies were analyzed and some implications for improving problem solving skills of first grade children were considered. Underhill considered the importance of accuracy and modeling for first graders in problem solving to be essential. Also, Underhill gave six recommendations for first grade instruction and an explanation of each one. The use of manipulative aids was highly advised by Underhill. A thorough explanation of transformational and nontransformational problems was given.
(Underhill is a professor of mathematics education in the department of curriculum and instruction at the University of Houston.)

Applications in the Junior High

Allen, Layman E., and Ross, Joan, Improving Skills in Applying Mathematics Ideas: A Preliminary Report on the Instructional Gaming Program at Pelham Middle School in Detroit, Educational Resources Information Center, ED 113 163, National Gaming Council, Pittsburgh, Pennsylvania, October, 1974, 13 pp.

The performance of 237 students in ten eighth grade mathematics classes was assessed to measure their ability to solve problems which involved computational skills and the recognition of the relevance of a particular mathematics idea which was absolutely indispensable to the solution of the problem. Both pre-tests and post-tests were administered to these ten classes. Certain effects of these data were measured by two specially designed tests. The results of this study concluded that students did better if specific procedures were taught to them which could help them apply known skills at various levels of difficulty. The remainder of the study gave many reasons for teachers to give students many opportunities for applying the skills learned in the mathematics curriculum.

Ames, Pamela, "Bring a Bike to Class," The Arithmetic Teacher 25:50-53, November, 1977.

Mathematical decision-making can be involved in problems involving bicycles, according to Ames. In fact, Ames believed that bicycles could provide a series of problem solving situations. This article was primarily devoted to the presentation of the various situations in which bicycles could be used in the problem solving process. Some areas considered were: frames of the bikes and tricycles; bicycle gears and multigears, and symmetry. The areas in mathematics which would be involved in the different parts of the bicycles and how these areas lent themselves to the problem solving process were discussed and illustrated. Ames found that the junior high students found this application appealing.
(Ames is a member of the faculty of the University of Chicago Laboratory Schools.)

Hanel, Ann Marie, "Problem Solving in Mathematics," The Arithmetic Teacher 29:25, May, 1982.

Hanel summarized the current ideas on problem solving in mathematics in the Lane Education Service District in Eugene, Oregon. Problem solving packets were mentioned which have been developed for grades four through nine to improve problem solving skills and to integrate problem solving with adopted textbooks. The instructional approach of the packets was discussed by Hanel. The information was given to promote the application of these skills in the classroom. The cost factors and availability of these packets were included in the latter part of this article.
(Hanel is employed at the Lake Washington School District.)

Hecht, Anne T., "Environmental Problem Solving," The Arithmetic Teacher 27:42-43, December, 1979.

Hecht devoted the entire article to describing in detail the use of the environment in solving mathematics problems. Students were given specific problems such as the height of a tree or the depth of a lake to solve by the end of the day. Several other examples were given by Hecht. The results of this method were that the students thoroughly enjoyed the process and later would compare methods and theories of approach with each other in the classroom. Approaching problem solving in this method would have the problem solving process become more meaningful to the students and an experience the students would not so easily forget.
(Hecht is an instructor at Kruse Elementary School in the Pasedena (Texas) Independent Schools.)

Lindquist, Mary Montgomery, "Let's Do It: Problem Solving With Five Easy Pieces," The Arithmetic Teacher 25:7-10, November, 1977.

Lindquist's article was primarily devoted to word problems that required a minimum amount of computation but often a maximum amount of thinking. These problems could be used at any time. They all used a set of geometric pieces. The topics involved were not only geometry but also included logic, combinations, division, and money. All of these topics were presented in the form of a game or puzzle rather than dealt with formally. The games and puzzles all involved five pieces. With these pieces, basic relationships and basic patterns were found. The bulk of the article was devoted to the specific applications of this concept with many diagrams to illustrate the main point of the activity.
(Lindquist is a professor at the National College of Education in Evanston, Illinois.)

Silver, Edward A., Problem-Solving Performance and Memory for Mathematical Problems: Solving Related Problems, Educational Resources Information Center, ED 175 630, Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, California, April 8-12, 1979, 26 pp.

This paper was written by Silver to examine a student's problem solving abilities and the student's recall of previously solved mathematics problems. Seventy-seven seventh grade students' perceptions of problem relatedness, ability for problem solving, and recall of information from previous problems were discussed. The results of Silver's study were that good and poor problem solvers differed in recall. Good problem solvers tended to recall information about a problem structure. Poor problem solvers rarely did. Good problem solvers recalled details of a problem's statement better than poor problem solvers. Significant transfer effect was found from solution and discussion of one problem's structure to the related problems.

Applications in High School

Cohen, Louis S., "Open Sentences - The Most Useful Tool in Problem Solving," The Arithmetic Teacher 14:263-267, April, 1967.

Cohen gave careful attention to the steps in the development of the ability of a student to translate a given situation into mathematical symbolism as a tool in problem solving. Cohen discussed and gave examples of number phrases, variables, word phrases, and number sentences. Little or no mechanics were mentioned in this article since the main emphasis was on the writing of mathematical sentences and not on their solution. (Cohen is a mathematics coordinator in grades 1 - 12 in the Bloomington Public Schools, Bloomington, Minnesota.)

Krulik, Stephen, and Reys, Robert E., Problem Solving in School Mathematics, pp. 228-241, 1980 Yearbook of the National Council of Teachers of Mathematics, Reston, Virginia, 1980.

These particular pages of the yearbook were devoted to an annotated bibliography by Sara F. Mason. The bibliography contained materials that were of primary interest to teachers of all grades--no research project were included. The entries were from books and journals. They were organized into six sections: 1) bibliographies, 2) general works, 3) suggestions for teaching, 4) puzzles and recreations, 5) mathematical discussions, and 6) collections of problems. Several selections in this chapter of the yearbook contained pertinent applications for high school students.

Morris, Janet Parket, "Problem-Solving With Calculators," The Arithmetic Teacher 25:24-26, April, 1978.

The calculator was mentioned as an invaluable tool in the problem solving process. The students in the problem solving situation should be concentrating on the thinking process involved rather than the computation involved. The calculator should be used as a tool to aid in the tedious computations. The calculator enabled a student to concentrate on the higher level aspects of problem situations. Therefore, Morris described several activities in which the calculator could be used to eliminate the tedious computations in problem solving. Famous problem solvers such as Polya would have advocated the technique of "guess and check". The calculator could reinforce this technique. Word Problems that are worked by the students could now have real-world numbers in them since the difficult calculations would be alieviated with this valuable tool. Using real-world numbers would create more interest on the part of the students.
(Morris is an assistant professor of mathematics education at the University of Michigan in Flint.)

Troutman, Andria Price, and Lichtenger, Betty Plunkett, "Problem Solving in the General Mathematics Classroom," Mathematics Teacher 67:590-597, November, 1974.

The authors described seven specific abilities which they felt were needed and were related to solving problems. The core of the article dealt with the suggestion of appropriate problem solving activities that the authors felt helped to develop these necessary abilities. The activities mentioned were described and examined thoroughly.

SUMMARY

Problem solving has been and will always be of concern to educators, mathematicians in particular. It can be seen from the research in this study that the topic of solving problems had been used by great mathematicians and philosophers as far back as 300 A.D. People such as Pappas, Descartes, Euclid, Apollonius of Perga, and Aristaeus were greatly involved in the discovery method of solving problems. Again in the forties, Polya devoted a great deal of thought to this process. Krulik, Suydam, Reys, and Rudnick came forward in the late seventies and early eighties. They have repeatedly stressed the fact that problem solving is not only pragmatic but essential in the mathematical curriculum.

Problems face each of us in our everyday lives. Situations arise in which we are personally involved, in which our goal is impeded by some obstacle, and we need to be able to analyze the various options we have at our disposal which would enable us to overcome these obstacles.

Cohen and Johnson have stated that problem solving may involve a number of activities on our part. Activities such as decision making, exploring, organizing, remembering, diagramming, formulating, generalizing, applying, and many others were mentioned by these authors. What good would it do us if we were faced with a problem and analyzed our way through to a proper procedure to solving it and then did not follow through and applied what we had learned to remove the problem from our lives. Follow-through is an important aspect of realistic problem solving!

Krulik was very effective in the presentation of what a problem really was and in the presentation of various strategies one could utilize to work to a logical solution. Suydam, on the other hand, always managed to present practical applications in our everyday teaching in the classroom and living. Moursund and Reys made us aware of the fact that we need to incorporate our technological advances in our treatment of problem solving since these advances are a realistic part of our lives. In respect to problem solving in the mathematical school curriculum, these advances would refer to the use of calculators and computers as instruments of facility.

As far as the implications of problem solving in the mathematics classrooms, Burton pointed out a very basic notion that the word problems in textbooks should actually reflect cultural aspects in which the students live in order to make the problems more relevant to them. Unless students are motivated to get involved in the thinking process, an essential element of good problem solving will be lacking.

It cannot be forgotten that Jean Piaget has influenced problem solving tremendously in relating the process of human development in the process of thinking and reasoning. We need to keep in mind that children begin on a concrete operational level and need to make a transition to the formal operational level. This concept should influence our presentation of materials in the classroom, especially in the junior high grades where this transition very often is taking place. Therefore, we need to incorporate the use of manipulative aids even at this level. It has often been thought that manipulative aids are strictly for the younger children where their need to relate to models is obvious. However, even though students do not openly demonstrate this need in the junior high level and higher, the need still exists. And we, as teachers, need to be fully aware of this need in order to be effective.

It is important to be aware of our physical make-up as human beings. This was brought out by Wheatley in the article concerned with the role of the right hemisphere of the brain in the problem solving process. In order to enhance problem solving, we need to address the right side of the brain more often in our presentations. We need to present problem situations in images often rather than limiting them to word problems on paper or in texts.

It is not the intent of this study to minimize the importance of the learning of computational skills which are extremely necessary. It is the object of this report to stress that the development of good problem solvers needs to come from an atmosphere where the thinking processes involved are of utmost importance. Perhaps, as Masse suggested, we need to stimulate the students' thinking processes by removing the "numbers" out of the problems. This way the students would be placed in a position where their attention would not be on the mathematically computed answer but rather on the learning process of analyzing the right approach to a solution. Thereby we would be able to stress the fact that the ability to analyze the problem situation is equally as important as the correct mathematical answer.

As we compare and contrast the recommendations of authors on the techniques and strategies presented, we can see that through the ages the approaches have a common basis with Polya's approach. The heuristic approach has been widely accepted by mathematicians. This approach has been found to be essential and effective regardless of grade level be it elementary, junior high, high school or even college. The only contrast that can be seen is that the type of problems and the topics covered in the problems that the children have been exposed to have changed a great deal.

We have seen that problems arise from our cultural, occupational, and environmental surroundings. But, the thinking process remains the same.

We must first accept the fact that a problem exists for us, we need to become personally involved, and we need to be able to analyze the situation by using our previous experiences as a basis and know how to use resources available to us in order to solve the problem. We also need to see that, after we have become involved, we periodically look back to see that we are staying on course and not straying from the ultimate goal we have set for ourselves.

As Suydam and Weaver have stressed, unless children are introduced and exposed to a number of alternative techniques and strategies, how could they ever have the option of choosing the approach they find most comfortable. The fact that there is one best method has been refuted by Suydam and many other authors. Unless teachers are themselves aware of the common pitfalls mentioned by Suydam, how could one expect to avoid them. We need to incorporate as many approaches in our classroom as we possibly can, trying to reach students at various levels of development. We need to keep in mind that through repeated applications, they will be able to grasp the thinking process and make this process their own.

As Burns proposed, in real life situations, we rarely are given all the information one needs in one tidy package. Therefore, it would be more practical to expose students to problems sporadically in which all avenues are not directly outlined for them. We need to make the classroom open to discussion on the various methods used by students and perhaps point out the advantages of some over others but at all times accept all logical and mathematically correct approaches.

One point brought out by Jenson and O'Neil which was felt to be important was that sometimes one must back away from a problem for awhile

in order to get a fresh look before proceeding. Students need to be aware of this fact. Keeping in line with this train of thought, it is important to mention Higginson's "functional fixedness" concept. We need to keep our thinking process and approaches more global in view. We need to teach students that many times they cannot see the solution to a problem because of limitations to one mathematical concept or theorem. By extracting as many concepts as possible from a problem, a better idea of how mathematics knowledge evolves may be seen.

In the 1950s an emphasis was on skill learning. And, in the 1960s this emphasis developed. Through the 1970s and early 1980s a back to basics approach was experienced. Through the years mathematics' content has grown. We do not always know what types of problems students will be facing ten, twenty, or thirty years from now. Therefore we should be teaching strategies rather than specific types of problems. Needs as well as clientele and times change.

CONCLUSIONS

We have really not rediscovered problem solving. Problem solving has always been an ultimate goal of each individual content, topic, and skill exercise. However, we are now coming to realize that problem solving should be one of our immediate goals. It must be a goal which directs daily activities. We should involve problems that incorporate higher level cognitive skills. Also, we are more fully aware of the fact that specific instructional activities, techniques, or strategies must be employed in pursuing problem solving abilities.

At every grade level in our mathematics curriculum there should be a structured and monitored instruction in such a way that regular growth toward problem solving subgoals occur. Every student from kindergarten on up must have the opportunity to grow in his/her ability to apply existing skills and knowledge to situations that are relevant to them without rehashing previous activities. The mathematics curriculum should be taking specific steps in meeting this challenge and need of the future.

RECOMMENDATIONS

It is recommended that:

1. Teachers need to understand that the thinking process in problem solving is of utmost importance. We need to present a proper approach to the problem solving process. A visual representation of this approach in given in Appendix A of this report.
2. Teachers need to understand that problem solving affects the lives of all individuals and permeates all aspects of society. A visual representation of problem solving techniques and their relationship to various aspects of society is given in Appendix B of this report.
3. Teachers need to understand that they have no definite means of preassessing the future needs of each of their students since each student is a unique individual. However, they can provide them with a logical and accurate thinking process for approaching problems in their lives. In order to accomplish this, they need to provide practical activities in the classroom which will enable students to make the transition from the classroom to real-life situations as smoothly as possible. This can be done by providing students with appropriate techniques and effective thinking processes.

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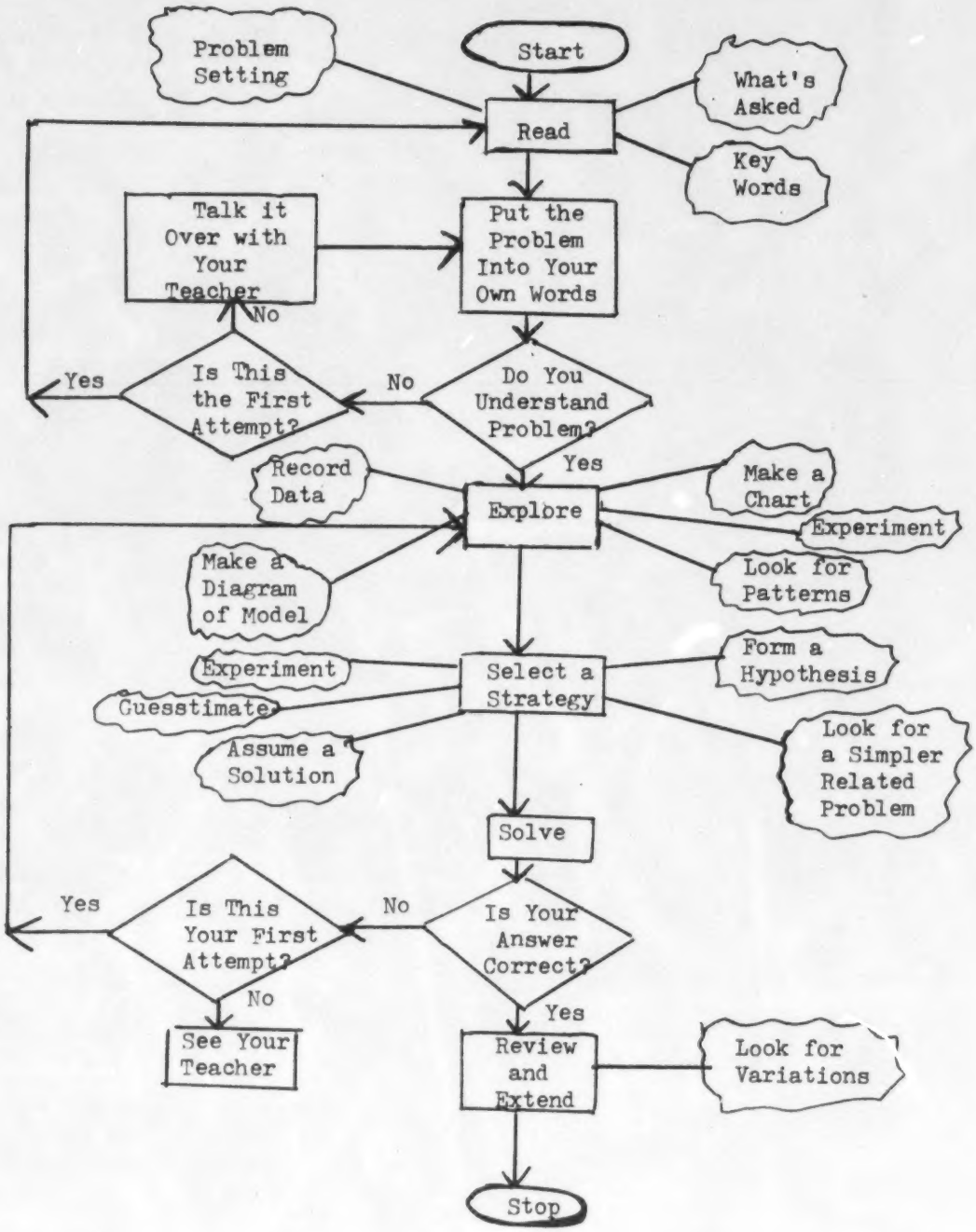
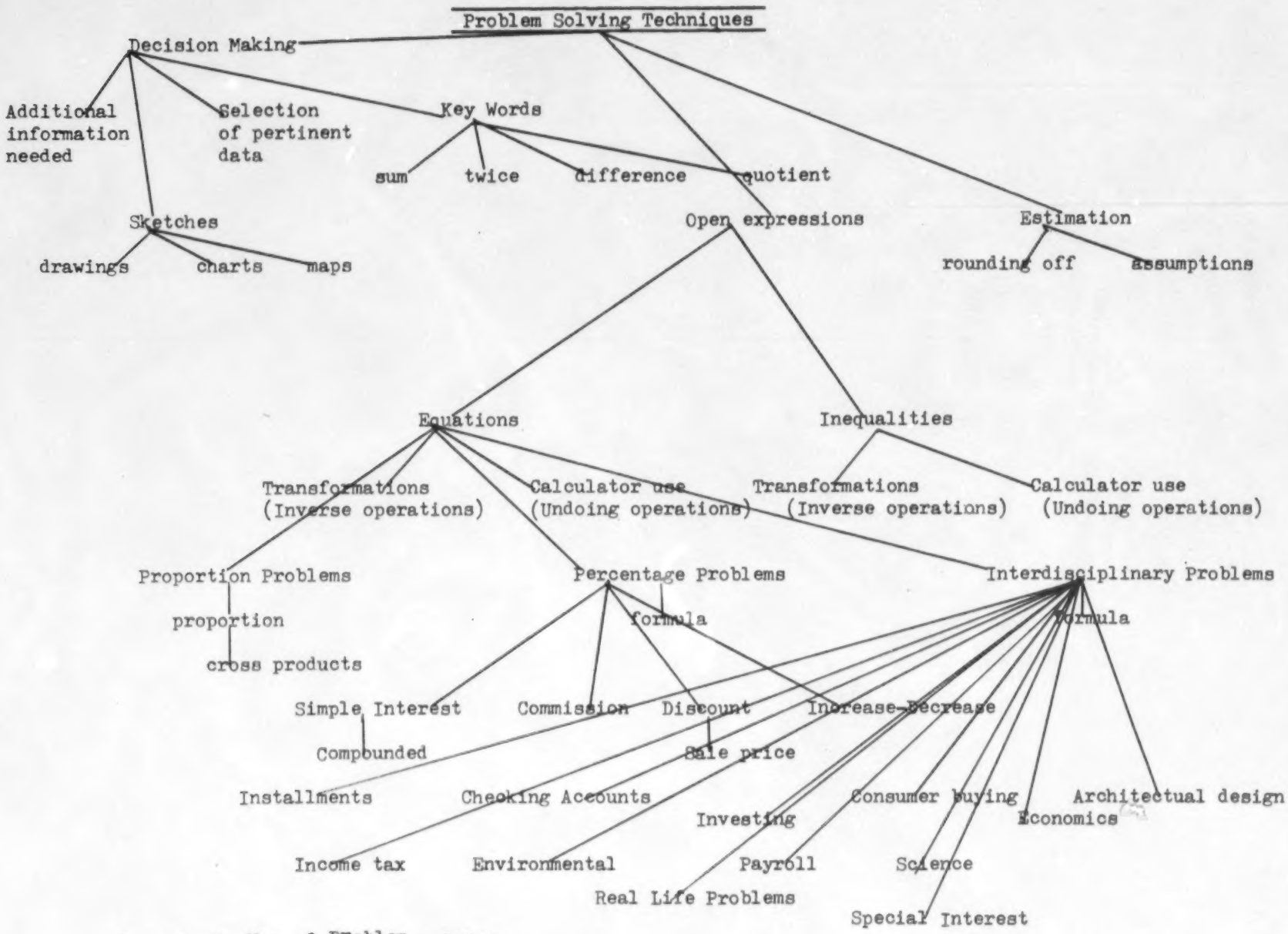
APPENDICES

APPENDIX A



Flow Chart of Problem Solving Approach
(Krulik and Rudnick, 1980:60)



Conceptual Map of Problem Solving

APPENDIX B